Tutorial: Inverse Transform Method

Question 1

Using the inverse transform method, write an R function to generate a random sample of observations from the following pdf,

$$f_X(x) = \begin{cases} 4x^3, & \text{if } 0 < x < 1 \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

Using the written function generate 1000 random numbers.

Draw a histogram and superimpose the density function (1) in the same graph.

Question 2

Using the inverse transform method, write an R function to generate a random sample of observations from the standard Laplace distribution with pdf,

$$f_X(x) = \frac{1}{2}e^{-|x|}, x \in \mathbb{R}. \tag{2}$$

Using the written function generate 1000 random numbers.

Draw a histogram and superimpose the density function (2) in the same graph.

Question 3

Using the inverse transform method, write an R function to generate a random sample of observations from the following pdf,

$$f_X(x) = \begin{cases} \frac{1}{x^2}, & \text{if } x > 1 \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

To test your R function, plot a histogram of 10000 random numbers together with the density function in 3.

Question 4

The CDF of the Pareto (a, b) distribution is

$$F_X(x) = 1 - \left(\frac{b}{x}\right)^a, x \geq b > 0,\ a > 0. \tag{4}$$

i) Using the inverse transform method, write an R function to generate a random sample of observations from the Pareto distribution.

ii) Generate 1000 random numbers from $Pareto(2, 2)$, $Pareto(1, 1)$.

iii) Draw histograms of the data you generated for each distributions in (ii). The two histograms should be on the same graph. Comment on any noticeable differences between the histograms.

Question 5

Using the inverse transform method, write an R function to generate a random sample of observations from the following pdf,

$$f_X(x) = \begin{cases} x, & \text{if } 0 \leq x < 1 \\ 2 - x, & \text{if } 1 \leq x \leq 2. \end{cases} \tag{5}$$

To test your R function, plot a histogram of 10000 random numbers together with the density function in 5.